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Research Note

Convective Envelopes and Radial Pulsation of Massive Red Supergiants

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A grid of convective envelope models covering a wide range of astrophysical parameters (including mass loss) has been constructed for M-type supergiants with the help of theoretical evolutionary tracks, mixing-length theory, and the inclusion of hydrogen and helium ionization zones. Linear adiabatic pulsation theory has been used to obtain the first four normal modes of radial pulsation for all the models. For the fundamental mode, the quantity $W = P(\mathcal{M}/\mathcal{M}_\odot)(R/R_\odot)^{-2}$ is found to be more nearly constant than is the usual quantity $Q = P(\mathcal{M}/\mathcal{M}_\odot)^{1/2}(R/R_\odot)^{-3/2}$, although the *overtone* Q values are practically constant. All the convective envelopes are found to be dynamically stable. The theoretical pulsation calculations confirm that the primary observed period in variable M-type supergiants is probably due to the fundamental mode of radial pulsation and that the long secondary period is not due to a normal mode of radial pulsation.

Key words: red supergiants — convective envelopes — pulsation

Previous work (Stothers, 1969, "Paper I"; Stothers and Leung, 1971, "Paper II") on the luminosities, masses, and periodicities of M-type supergiants has relied on rough theoretical and empirical values of the pulsation constant Q. The present paper attempts to improve the theoretical determination of the pulsation constants for convective envelopes of massive stars.

The model envelopes calculated in Paper I were based on a number of very crude approximations. Improved models will be calculated here according to the prescription given by Iben (1963, 1965). Radiation pressure is included fully as before, but the state of the gas is now computed for hydrogen and helium in the stages H_2, H, H^+, He, He^+, and He^{++}, as well as a hypothetical metal in the stages M, M^+ (with an ionization potential equal to 7.5 eV). Convection is treated by Böhm-Vitense's (1958) version of the mixing length theory with the ratio of mixing length to density scale height, α, left as an adjustable parameter. Thus, the superadiabatic region near the stellar surface is taken roughly into account, while the surface boundary condition is adapted from Eddington's (1926) approximate solution for a radiative atmosphere. Opacities due to Cox and Stewart (1965), in the form given by Stothers and Simon (1970) as an improvement of Christy's (1966) formula, have been used for temperatures above 4000° K, while the opacities of Kippenhahn *et al.* (1958) have been used below 3000° K; a temperature-weighted mean has been adopted in between. Use of the new opacity tables of Cox and Stewart (1970) for temperatures above 4000° K has been found to make only a small change in our envelope models. The bottom of the convective envelope is defined, in the usual way, as the point at which the radiative and adiabatic temperature gradients become equal.

Fundamental and overtone periods of radial pulsation of the model envelopes have been calculated as in Paper I[1]) by integrating the mechanical wave equation for small adiabatic pulsations, neglecting turbulent pressure, viscosity, and any interaction between the convection and pulsation (Ledoux and Walraven, 1958, p. 458). A conventional standing-wave boundary condition is used at the stellar surface (Ledoux and Walraven, 1958, p. 458), whose precise location, although ambiguous for distended atmospheres (Auman, 1969; Keeley, 1970b), is not critical for determining the pulsa-

[1]) Two typographical errors in Paper I should be pointed out. The right-hand side of the definition of A in Eq. (5) should be multiplied by the radiation density constant a, and the right-hand side of Eq. (8) should read $(3\pi/G\omega^2)^{1/2}$.

tional eigenvalues. It should be noted that we are not testing the envelopes for actual pulsational instability but are only deriving the periods of radial pulsation, P. The usual pulsation constant

$$Q = P\,(\mathcal{M}/\mathcal{M}_{\odot})^{1/2}\,(R/R_{\odot})^{-3/2}$$

refers to the bulk properties of the whole convective envelope. Alternatively, a quantity

$$W = P\,(\mathcal{M}/\mathcal{M}_{\odot})\,(R/R_{\odot})^{-2}$$

is the natural form of the pulsation constant if the oscillations are confined to the upper layers of the envelope (e.g. Rosseland, 1949; Gough *et al.*, 1965).

A grid of model envelopes has been constructed covering a broad range of masses and luminosities for red supergiants in the phases of helium burning and carbon (or oxygen) burning in the core. Published evolutionary sequences for stars of 9, 15, 20, 30, and 60 $\mathcal{M}_{\odot}$ have furnished best estimates of the luminosities, chemical compositions, and envelope masses (Hayashi *et al.*, 1962; Iben, 1966a, b; Stothers, 1966; Hofmeister, 1967; Stothers and Chin, 1968, 1969; Chiosi and Summa, 1970; Paczynski, 1970). For consistency, deep convective envelopes extending down to the hydrogen-burning shell have been adopted. But various changes in the envelope parameters have then been imposed, such as a reduced envelope mass due to extensive mass loss (the model of 7 $\mathcal{M}_{\odot}$ based on an original mass of 20 $\mathcal{M}_{\odot}$), with the assumption being made that the luminosity and core mass of the original model would not be changed significantly; this is probably an acceptable assumption for our purposes. In all, we have tested the sensitivity of Q to the following parameters: mass ($\mathcal{M}$), luminosity (L), opacity ($\varkappa$), ratio of mixing length to density scale height (α), mass fraction contained in the convective envelope ($\mathcal{M}_{\text{conv}}/\mathcal{M}$), hydrogen-to-helium ratio (X/Y), metals abundance (Z), and mass loss.

Table 1 lists the basic model characteristics adopted from the published evolutionary tracks.[2]) Revised effective temperatures, however, have been derived in the present work since these are determined by the assumed values of α and opacity. Our adopted range of α runs from small mixing lengths ($\alpha \ll 1$) up to a mixing length formally equal to twice the total depth of a typical convective envelope ($\alpha \approx 20$). With the opacities described above, the effective temperatures turn out to be considerably higher than those observed ($\log T_e = 3.54$ for an average spectral type of M2 according to Lee, 1970) unless $\alpha \ll 1$. An artificial increase in opacity has therefore been introduced in the important temperature regime below 10000° K (the ionization temperature of hydrogen) in order to achieve approximate agreement between theoretical and observed effective temperatures for the case $\alpha = 1$. Alternatively, one might proceed in the usual fashion and reduce α to an appropriate value, say < 0.5 (as in fact has been suggested by Iben for main-sequence stars). Both procedures mask in a crude way all the uncertainties in the mixing-length theory. Fortunately, they are found not to have a large effect on the radial pulsation constants for most of the models. On the basis of the arbitrarily modified opacities as well as the unmodified opacities, effective temperatures and radial pulsation constants for the models of Table 1 are given in Table 2. It should be noted that the observationally most relevant models are the core helium-burning models for 9, 15, 20, and probably 30 $M_{\odot}$. Additional integrations have been performed for the low-luminosity model of 15 $\mathcal{M}_{\odot}$ with various assumed chemical compositions. A moderate increase in hydrogen or metals content was found to reduce T_e slightly but to have no significant effect on Q or W. In general, T_e is also reduced and Q_0 is increased by increasing $L/\mathcal{M}$ (due to evolutionary brightening, mass loss, or simply a higher initial mass), $\mathcal{M}_{\text{conv}}/\mathcal{M}$, or $\varkappa$; or by decreasing α (cf. Kippenhahn *et al.*, 1958; Hayashi *et al.*, 1962; Keeley, 1970a). In none of our envelope models does dynamical instability

[2]) Strictly speaking, the envelope model for 30 $\mathcal{M}_{\odot}$ during carbon burning should have a somewhat larger helium and metals abundance, due to the occurrence of deeper convective mixing in this phase than during helium-burning (Stothers and Chin, 1969).

Table 1. *Basic parameters for the theoretical models of convective envelopes*

$\mathcal{M}/\mathcal{M}_{\odot}$	X	Z	$\log (L/L_{\odot})$	$\mathcal{M}_{\text{conv}}/\mathcal{M}$	Core burning phase
60	0.500	0.030	6.0	0.6	He, C
30	0.600	0.030	5.5	0.6	He, C
30[a])	0.600	0.030	5.5	0.3	He
20	0.563	0.044	5.0	0.7	He
15[b])	0.600	0.030	5.0	0.8	C
15	0.600	0.030	4.6	0.8	He
9	0.690	0.020	4.0	0.8	He, C
7[c])	0.563	0.044	5.0	0.15	He

[a]) Small mass fraction in the convective envelope.
[b]) Bright luminosity.
[c]) Remnant of initially 20 $M_{\odot}$.

Table 2. *Effective temperatures and radial pulsation characteristics of the theoretical models of convective envelopes*

α	$\varkappa$	$\mathcal{M}/\mathcal{M}_\odot$	$\log T_e$	Q_0 (day)	W_0 (day)	P_0/P_1	α	$\varkappa$	$\mathcal{M}/\mathcal{M}_\odot$	$\log T_e$	Q_0 (day)	W_0 (day)	P_0/P_1
0.4	unmod.	60	3.61	0.130	0.022	3.4	1	mod.	9	3.57	0.058	0.011	1.8
0.4	unmod.	30	3.58	0.108	0.016	3.0	1	mod.	7[c])	3.51	0.125	0.010	3.8
0.4	unmod.	30[a])	3.61	0.085	0.014	2.6	2	mod.	30	3.54	0.085	0.012	2.2
0.4	unmod.	20	3.56	0.088	0.014	2.4	2	mod.	15	3.58	0.065	0.012	1.8
0.4	unmod.	15[b])	3.53	0.101	0.013	2.8	2	mod.	9	3.60	0.057	0.012	1.7
0.4	unmod.	15	3.57	0.077	0.013	2.1	2	mod.	7[c])	3.51	0.118	0.010	3.6
0.4	unmod.	9	3.57	0.064	0.012	1.9	20	mod.	30	3.55	0.077	0.011	2.0
0.4	unmod.	7[c])	3.56	0.119	0.011	3.8	20	mod.	30[a])	3.56	0.072	0.011	2.0
1	mod.	60	3.53	0.127	0.018	3.2	20	mod.	20	3.56	0.065	0.010	1.7
1	mod.	30	3.53	0.097	0.013	2.6	20	mod.	15	3.59	0.061	0.011	1.7
1	mod.	15[b])	3.53	0.085	0.011	2.3	20	mod.	9	3.61	0.051	0.011	1.7
1	mod.	15	3.55	0.068	0.012	1.9	20	mod.	7[c])	3.52	0.107	0.009	3.4

[a]), [b]), [c]) see Table 1.

develop (Q_0 remains real), even for extensive loss of mass. This confirms and extends the earlier results of Paczynski and Ziolkowski (1968).

Detailed examination of our models indicates that the larger values of Q_0 are attributable chiefly to the reduction of Γ_1 by the second ionization of helium inside a deep, thick zone. The first ionization of helium and the dissociation and ionization of hydrogen occur in narrower zones closer to the surface, and therefore have less effect on Q_0, which is determined mostly by the physical conditions near a radius fraction of 0.7 (Epstein, 1950). Our models show that Γ_1 attains minima well below 4/3 (the minimum for a mixture of fully ionized gas and radiation) at radius fractions of typically 0.95, 0.90, and 0.80, in the ionization zones of H, He, and He^+, respectively. Therefore, in many of our models, even the ionization zone of He^+ lies too near the surface to influence Q_0 significantly. Furthermore, the superadiabatic part of the envelope is also rather small, covering a mass fraction of only 0.05–0.10. As a consequence, the physically interesting values of Q_0 in Table 2, viz. Q_0=0.06–0.08 day, tend to be very close to those derived in Paper I by assuming adiabatic envelopes and complete ionization of the gas.

A somewhat more stable quantity than Q_0 is W_0. Its most likely value is $W_0 = 0.012 \pm 0.002$ day, independently of the uncertainties in $\varkappa$ and α, of the evolutionary stage of the star, or of whether mass loss has occurred or not, which is not the case for Q_0.

Overtone values of Q are remarkably uniform. This is apparent also by a glance at the tabulation of Ledoux and Walraven (1958, p. 473) containing models with widely different physical properties. All of our models have $Q_1 = 0.030 - 0.040$ and $Q_2 = 0.018 - 0.024$ day. Ratios of the periods are, in all cases, $P_0/P_1 = 1.7 - 3.8$, $P_1/P_2 \approx 1.6$, and $P_2/P_3 \approx 1.3$; but the physically realistic cases have $P_0/P_1 \approx 2$.

Generally speaking, the massive red supergiants have smaller Q_0 values than do the red giants of small mass (Paczynski and Ziolkowski, 1968; Keeley, 1970a, b). This is primarily due to the hotter effective temperatures and "normal" luminosities for their masses, in the case of the red supergiants. Keeley's work on low-mass red giants is of interest here because he performed full, nonlinear hydrodynamical calculations. The main advantage of such a treatment over linearized adiabatic theory is the possibility of determining which modes are actually excited. Keeley found that, in low-mass stars at least, the first overtone is preferred when only a small mass fraction of the star lies above the hydrogen ionization zone, whereas the fundamental mode becomes stronger when the latter zone resides at a deeper level. But it is not yet known (theoretically) which mode is preferentially excited in the massive supergiants.

The semiempirical luminosities and masses of variable red supergiants derived in Paper II on the assumption of radial pulsation with $Q = 0.06$ day were found to be in substantial agreement with the available empirical results, and are not significantly changed by using the new theoretical Q_0 or W_0 values calculated in this paper. However, as already explained, W_0 is a better pulsation constant to use in future work. The main conclusions based on the present work can be summarized briefly. First, radial pulsation in the fundamental mode is confirmed as the likely explanation of the primary period observed in variable M supergiants; adoption of even the first overtone ($Q = 0.035$ day) would brighten the semiempirical absolute magnitudes by 0.9 mag. Second, if the secondary period were due to the fundamental mode and the primary period to an overtone (as could not be ruled out in Paper II), then our present theoretical calculations would predict $P_0/P_1 \approx 2$, $P_0/P_2 \approx 3$, and $P_0/P_3 \approx 4$. Even though the fundamental mode and first overtone are the only modes likely to be actually excited (Keeley, 1970a, b), all of these theoretical period ratios fall far short of explaining the observed period ratios, whose average value is ~ 8. If one were to argue that most of the variable M supergiants are somehow close to the limit of dynamical stability (so that $Q_0 \to \infty$, $Q_1 \approx 0.035$, and hence $P_0/P_1 \to \infty$), one would then expect to see a large increase in the observed period ratios with a small increase in spectral subtype; the observational data (Paper II) contain no indication whatsoever of such a trend. Hence the secondary period does not seem to be due to a normal mode of radial pulsation.

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